by constructing the orthogonal complement of N (all vectors in N are gradients of scalar functions) in v'. If $\mathbf{p} = \nabla v \epsilon N$ and $\mathbf{q} \epsilon N\perp'$, then

$$0 = \int_{S_o} \mu\, \mathbf{q}\cdot\nabla v\, dA =$$

$$-\int_{S_o} \mu\, v \nabla\cdot\mathbf{q}\, dA + \int_{S_o} \mu v\, \mathbf{q}\cdot\vec{dl} =$$

$$-\int_{S_o} \mu\, v\nabla\cdot\mathbf{q}\, dA$$

Consequently $N\perp$ has been enlarged to

$$N\perp' = [\mathbf{q} : \nabla\cdot\mathbf{q} = 0 \text{ in } S_o]$$

by invariant imbedding and

$$N = [\mathbf{p} : \mathbf{p} = \nabla v, v = 0 \text{ on walls of duct}]$$

Note that $N\perp'$ still includes the previous set $N\perp$.

The previous upper and lower bounds for the volumetric flow rate become

$$\frac{D'^2(\mathbf{q}_o, \nabla v)}{\mathcal{P}\, D'(\nabla v, \nabla v)} \leq Q \leq \frac{1}{\mathcal{P}} D'(\mathbf{q}_1, \mathbf{q}_1)$$

$$\mathbf{q}_1 - \mathbf{q}_o \epsilon N\perp' \qquad (14)$$

where $v\epsilon N$ and $\mathbf{q}_o(x, y)$ is any function satisfying

$$\nabla\cdot\mathbf{q}_o = \frac{-\mathcal{P}}{\mu}$$

FUNCTION SPACE EQUIVALENT OF DUAL VARIATIONAL STATEMENTS

In this section the bounds for the volumetric flow rate which were obtained by Stewart (2) are viewed in function space. In so doing the correspondence between the method of dual variational statements and the geometrical constructions in a function space becomes apparent.

From the definition of $\mathbf{q}_o$ and $v\epsilon N$ it follows that

$$D'(\mathbf{q}_o, \nabla v) = \int_{S_o} \mu\, \mathbf{q}_o\cdot\nabla v\, dA =$$

$$-\int_{S_o} \mu\, v\nabla\cdot\mathbf{q}_o dA = \mathcal{P}\int_{S_o} v\, dA$$

and consequently (15)

$$2D'(\nabla v, \mathbf{q}_o) - D'(\nabla v, \nabla v) =$$

$$2\mathcal{P}\int_{S_o} v\, dA - \mu \int_{S_o} \nabla v\cdot\nabla v\, dA \qquad (16)$$

Because of Equation (16) and the definition of $\mathcal{P}$, the bound (11) for the special case $\alpha = 1$ yields

$$(p_1 - p_o)\int_{S_o} v\, dA + \frac{\mu L}{2}\int_{S_o} \nabla v\cdot\nabla v\, dA \geq$$

$$\frac{-(p_o - p_1)}{2} Q \geq -\frac{L}{2} D'(\mathbf{q}_1, \mathbf{q}_1) \qquad (17)$$

which is identical to Stewart's statement of bounds for $J = \frac{-1}{2}(p_o - p_1)\, Q$. It was pointed out above that inequalities (14), not (11) with $\alpha = 1$, give the narrowest possible bounds. Thus an immediate reward for viewing the problem in a function space is finding that Stewart's lower bound for the volumetric flow rate is not the best possible; however it does simplify numerical computations.

From the above reconstruction of Stewart's bounds it is apparent that dual variational statements represent statements about the projections of function-space vectors onto specific hyperplanes in function space. The utility of function space methods is twofold. Firstly, they afford a clear picture of the approximation and narrowness of the bounds, and secondly, they can be extended to yield bounding techniques for the pointwise value of a function. Moreover, they are, in the words of Synge (3, pp. 33 to 34), "in tune with physical space intuitions which are sometimes indispensable for discovery and rapid understanding."

NOTATION

dA	= differential element of area
D	= integral defined by (3)
L	= axial distance
L	= periphery of duct cross section
p	= value of thermodynamic pressure referred to a given datum
$\mathcal{P}$	$= \frac{p_o - p_1}{L}$
Q	= volumetric flow rate through duct
S_o	= cross-sectional area of duct
w	= local value of axial component of velocity field
∇	= two dimensional gradient $\left(\frac{\partial}{\partial x}\check{i} + \frac{\partial}{\partial y}\check{j}\right)$
∇^2	= two dimensional Laplacian $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$
μ	= viscosity

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Flow of Liquids in Horizontal Capillary Tubes

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Behavior of liquid-gas systems in the absence of gravitational forces presents interesting problems for the engineer who must design equipment for space flight. Utilization of surface tension forces which are independent of gravity to control the position of the liquid has considerable promise. Flow of several liquids in horizontal glass capillary tubes has been studied in order to predict flow under zero gravity conditions.

Prime purpose of the study was to determine whether or not capillary tubes could be used to pump liquid oxygen from a mass of liquid. The capillary could be either a conventional tube or a porous material. If a capillary tube is placed in a horizontal position, then it may be assumed that the gravity forces acting on a liquid flowing in the tube may be negligible. Based on this assumption it may be speculated that the behavior of a liquid in a capillary tube which is in a zero gravity environment can be estimated by studying the flow of liquids in horizontal capillary tubes.

PREVIOUS ANALYSIS

During initial considerations of the surface tension properties of LOX (liquid oxygen), it was thought that the rate of movement of LOX in a horizontal capillary could be calculated from known data. It was postulated that, under zero gravity conditions, the LOX would be drawn into the capillary until the tube would be filled or the surface tension forces would be balanced by a contrary force such as the viscous drag on the liquid.

(*Continued from page 278*)

INFORMATION RETRIEVAL

Key Words: Axial Diffusion-1, Axial Dispersion-1, Axial Mixing-1, Differential Equation-1, Laplace Transform-1, Mixing-Cell-1, Slug Flow-1, Amplitude Ratio-2, Frequency Response-2, Magnitude Ratio-2, Phase Shift-2, Transfer Function-2, Air-5, Water-5, Flow Rate-6, Frequency-6, Amplitude Ratio-7, Frequency Response-7, Phase Shift-7, Absorption-8, Distributed-Parameter System-8, Dynamics-8, Unsteady State-8, Carbon Dioxide-9, Linear Valve-10, Packed Column-10, Packed Tower-10, Sinusoid Generator-10, Thermal Conductivity Cell-10.

Abstract: The dynamic behavior of a packed-column gas absorber was investigated by comparing results of experimental frequency response tests with theoretical responses for three flow models: slug, mixing-cell, axial diffusion. Carbon dioxide was absorbed in water from air-carbon dioxide mixtures in a 6-in. I.D. Pyrex column packed to a depth of 5.12 ft. with ⅝-in. ceramic Raschig rings. A linear valve and two high-speed thermal conductivity cells of special design were employed. Observed discrepancies between theory and experiment are discussed.

Reference: Gray, Robert I., and John W. Prados, **A.I.Ch.E. Journal, 9,** No. 2, p. 211 (1963).

Key Words: Heat Transfer-8, Burnout-8, Convection-8, Forced-, Boiling-9, Flow-9, Shear-9, Stresses-9, Perimeter-6, Pressure-6, Fluxes-6, Flow Rates-6, Quality-7, Burnout-7, Boilers-10, Heat Exchangers-10, Exchangers-10, Rods-10.

Abstract: A new concept is presented for predicting burnout conditions for forced convection of boiling water in fuel elements of nuclear boiling reactors. The concept states the importance of considering the ratio of heated channel perimeter to total channel perimeter. Experimental data have been obtained from a study of burnout conditions in rod clusters at various combinations of the following variables: steam quality (0.01 to 0.52), pressure (2.5 and 10 kg./sq. cm.), surface heat flux (50 and 120 W./sq. cm.), and mass flow rate (0.03 and 0.33 kg./sec.). The results are compared with previous data for round ducts in terms of the perimeter ratio concept. The importance of surface shear stress distribution and the position of maximum shear stress in predicting burnout conditions is discussed.

Reference: Becker, Kurt M., **A.I.Ch.E. Journal, 9,** No. 2, p. 216 (1963).

Key Words: Diffusivity-8, Permeability-8, Solubility-8, Iron-9, Hydrogen-9, Time Lag-10.

Abstract: The rate of permeation of hydrogen through a highly purified iron (Ferrovac E) was measured over a pressure range of 1/30 to 300 atm. and a temperature range of 126° to 693°C. The effect of cold working of the annealed metal and of hydrogen purity were also studied. Permeability (ϕ), diffusivity (D), and solubility were obtained from the data.

Reference: Bryan, W. L., and B. F. Dodge, **A.I.Ch.E. Journal, 9,** No. 2, p. 223 (1963).

Key Words: Enthalpy-8, Hydrogen Bonding-8, t-Butanol-1, Benzene-1, Water-1, Solutions (Binary)-2, Temperature-6, Pressure-6, Composition-6, Enthalpy-7, Charts-7, Calorimeter-10, Adiabatic-, Flow-.

Abstract: Enthalpies of binary mixtures of 50 mole % t-butanol in benzene and 75 mole % t-butanol in water were measured in an adiabatic flow calorimeter at temperatures from 250°F. to 550°F. and pressures from 20 to 1,000 lb./sq. in. abs. Pressure-enthalpy charts with an estimated accuracy of ±1% are presented for these highly nonideal systems.

Reference: Shannon, Paul T., and Patrick S. O'Neill, **A.I.Ch.E. Journal, 9,** No. 2, p. 229 (1963).

(*Continued on page 282*)

Since the flows of interest are laminar or in the viscous range, it was postulated that the flow resistance could be calculated from Poiseuille's law. However a theoretical and literature investigation (*1* to *10*) of the parameters involved did not result in any correlation which can be used to predict the flow rate. Most of the experimental data in the literature were correlated with the derived equations, and the data covered a wide range of time and distance of capillary flow. For short durations (less than 10 sec.) and short flow distances (less than 5 cm.), the theoretical equations give, at best, only a fair approximation of the experimental flow rate of such liquids as LOX.

In the literature, all of the most recent correlations are derived by taking into account the rate of change of momentum of the contents of the capillary, the forces due to surface tension, viscous resistance, gravity, and end drag effects. Most of the authors concede that an exact treatment of the flow of fluid in a vertical or horizontal capillary due to surface tension would be difficult. In fact, an exact hydrodynamical treatment of the subject has not been attempted, since neither the hydrodynamic problem of accelerated flow nor the physicochemical phenomenon of the rate of wetting is well understood. In the correlations, the following simplified assumptions are made: the same forces act on accelerated and stationary liquid, wetting is very rapid, and Poiseuille's law holds. Most of the theoretical work has been devoted to vertical or inclined capillaries and not to horizontal capillaries which more nearly represent conditions of zero gravity. Many of the authors

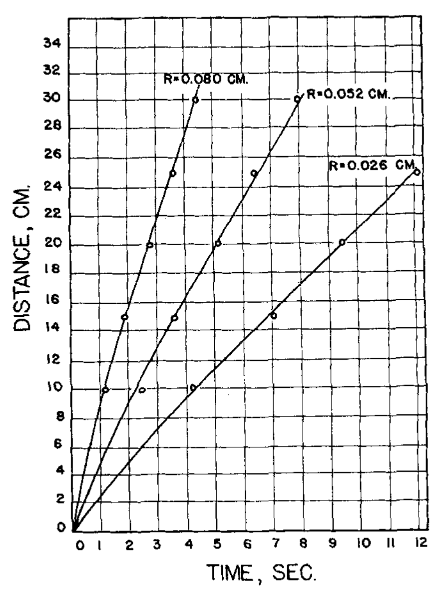

Fig. 1. Observed flow of liquid oxygen in horizontal glass capillaries.

extrapolated their final results to the horizontal flow condition by assuming that the contact angle is zero, and that the acceleration due to gravity is zero. Such assumptions yield approximate solutions to the basic differential equations of the motion of the fluid.

In order to understand better the phenomena of viscous flow in horizontal capillary tubes, theoretical and experimental studies were made of the flow of such liquids as LOX, water, and alcohol in glass capillaries. The theoretical correlation of the data is based on the use of dimensional analysis, and the experimental data were obtained with the use of horizontal, flat, spiral glass capillaries.

TABLE 1. FLOW OF LOX IN HORIZONTAL GLASS CAPILLARY TUBES

(TEMPERATURE —183°C)

Distance, cm.	Time, sec. Radius, cm. 0.080	0.052	0.026
	Experimental values		
10	1.2	2.5	4.2
15	1.9	3.7	7.0
20	2.8	5.2	9.5
25	3.7	6.5	12.2
30	4.5	8.0	15.0
	Calculated values*		
10	1.3	2.1	4.3
15	1.9	3.3	6.7
20	2.8	4.5	9.3
25	3.6	5.7	11.9
30	4.5	7.0	14.5

* From Equation (5).

EXPERIMENTAL

Experimental studies consisted of timing the flow of fluids in flat, horizontal, spiral capillary tubes, which were placed on the surface of the liquid. This was done either by slowly lowering the capillary until it touched the liquid, or the tube was placed on the surface and the liquid flow regulated by pressure at the exit end. At the instant the liquid entered the tube, the time measurement was started.

For some liquids, stopwatch timing was not sufficiently accurate, and an Esterline-Angus wattmeter recorder operated at a constant chart speed was used. A tapping key was provided to make rapid contact. As the liquid entered the tube the key was tapped, and this was repeated as the liquid reached each of the 5-cm. marks along the capillary. During the experiment the liquid was contained in a Dewar which kept it at a constant temperature. For the LOX, another flask containing LOX was placed inside the Dewar, which was also filled with LOX. This resulted in a nonboiling level surface of LOX in the inner container.

DIMENSIONAL ANALYSIS

With the use of the Buckingham π theorem (*11, 12*) the following dimensional groups were obtained:

$$\frac{\sigma T}{\mu r}, \frac{\mu T}{\rho r^2}, \frac{\sigma \rho r}{\mu^2}, \frac{Z}{r}$$

The method by which this correlation was substantiated is as follows. For a fixed temperature and tubes of three different sizes, a log plot was made of Z/r vs. $\left(\frac{\sigma T}{\mu R}\right)$, resulting into three parallel lines. The constant temperature resulted in constant density, surface tension, and viscosity. Hence, from a few experiments, the value of a is easily found. The values of the constants b and K can be determined from the following relation. Defining I as the value of intercept one obtains

TABLE 2. EXPERIMENTAL CONSTANTS IN THE FORMULA $\frac{Z}{r} = K\left(\frac{\sigma\rho r}{\mu^2}\right)^b \left(\frac{\sigma T}{\mu r}\right)^a$

$r = 0.026$ to 0.080 cm.
$Z = 0$ to 30 cm.

Fluid	a	b	K	Temperature, °C.
Liquid oxygen	0.90	0.848	7.405×10^{-8}	—183
Liquid nitrogen	0.87	1.65	0.246×10^{-10}	—195.8
Water	0.60	0.342-0.002 C	0.00708	55-40
Benzene	0.60	0.370-0.001 C	0.00794	7-40
Isopropyl alcohol	0.55	0.760-0.006 C	0.00794	0-40
Ethylene glycol	0.46	0.320-0.009 C	0.8534	0-20

If one chooses to correlate the data using the following groups

$$\phi\left[\frac{Z}{r}\ \frac{\rho\sigma r}{\mu^2}\ \frac{\mu T}{\rho r^2}\right] = 0 \quad (1)$$

then one of the more recent equations for the flow of fluid in a horizontal capillary may be used:

$$\left(\frac{Z}{r}\right)^2 = \frac{\cos\theta}{16}\left(\frac{\rho\sigma r}{\mu^2}\right) \left[\left(\frac{8\mu T}{\rho r^2}\right) + \exp\left(-\frac{8\mu T}{\rho r^2}\right) - 1\right] \quad (2)$$

This may be described as one of the forms of Equation (1). However it did not correlate the data for the experimental flow of LOX for short distances or short initial time periods.

A function which is often used to correlate dimensional groups is of the following form:

$$\left(\frac{Z}{r}\right) = K\left(\frac{\rho r}{\mu^2}\right)^b \left(\frac{\sigma T}{\mu r}\right)^a \quad (3)$$

Equation (3) is the best grouping found in this study. Other groupings did not result in any simple correlation.

$$\log I = \log K\left(\frac{\sigma\rho r}{\mu^2}\right)^b$$

or

$$\log I = b\log\left(\frac{\sigma\rho r}{\mu^2}\right) + \log K \quad (4)$$

A linear plot of $\log I$ vs. $\log\left(\frac{\sigma\rho r}{\mu^2}\right)$ can be used to find b and K.

From these plots one can determine all the constants in Equation (3). However experimental studies revealed that b is affected linearly by the temperature. For a given liquid at three different temperatures (0° to 55°C.), one obtains three lines which intersected at one point. The effects of temperature for LOX constant were not investigated.

From these graphs, which are based on theoretical and experimental work, the final equation on the initial flow of a liquid in a horizontal capillary can be expressed in the following form:

$$\frac{Z}{r} = K\left(\frac{\sigma\rho r}{\mu^2}\right)^{b+dC} \left(\frac{\sigma T}{\mu r}\right)^a \quad (5)$$

(*Continued from page 280*)

INFORMATION RETRIEVAL

Key Words: Control-8, Dynamics-7, Nonlinear-8, Reactors-9, Optimization-8, Computer-10, Constraints-8, Phase-Plane-9, State-Variables-6.

Abstract: For a linear or linearized stationary system, the optimum control action for the particular optimal function which is minimized can be generated by the technique of dynamic programming. Since a chemical reaction system can not always be approximated by a linearized model, a strategy for generating the optimal dynamic control action in the nonlinear case is developed. The application of the strategy is limited to physical systems which are stationary and bounded in the range of their variables. The computational scheme developed here, called an ***optimum predictor-controller,*** is essentially a switching device which determines when the control action switches.

Reference: Grethlein, H. E., and Leon Lapidus, **A.I.Ch.E. Journal, 9,** No. 2, p. 230 (1963).

Key Words: A. Mixture Enthalpy-8, Partial Enthalpy-8, Hydrocarbons-8, Methane-8, Propane-8, Binary-, Temperature-6, Pressure-6, Composition-6. B. Comparison-9, Mixture Enthalpy-8, PVTy-10, Equation of State-10, Redlich-Kwong-, Benedict-Webb-Rubin-. C. Correlation-8, Mixture Enthalpy-8, Partial Enthalpy-8, Pitzer-10, Hydrocarbons-8, Methane-8, Binary-.

Abstract: The isothermal effects of pressure and composition on the mixture and partial enthalpy differences of the methane-propane binary were derived from PVTy data. A graphical and numerical technique was used to derive the enthalpy differences from 100° to 460°F. and 200 to 2,000 lb/sq. in. abs. for the complete composition range of the binary.

The derived mixture enthalpy differences were compared with enthalpy differences that were calculated using the Redlich-Kwong and the Benedict-Webb-Rubin equations of state. The derived enthalpy differences were combined with literature data to give a modified Pitzer correlation of the enthalpy differences of methane-light hydrocarbon binaries.

Reference: Edmister, Wayne C., and Lyman Yarborough, **A.I.Ch.E. Journal, 9,** No. 2, p. 240 (1963).

Key Words: A. Water-1, Steam-1. B. Water-1, Steam-1, Pressure Drop-2, Flow-2. C. Elevated Pressure-3, High Velocity-3. D. Flow-2, Quality-4, Diameter-5, Length-5, Mixing-6, Surface Tension-7. E. Critical Flow-8, Pressure Drop-2. F. Critical Flow-8, Metastability-8. G. Pipe-9, Annulus-9, Probe-9.

Abstract: Two-phase critical flow of steam water mixtures was studied in concentric annuli. Pressure taps on the center rods permitted a study of the pressure profile both upstream and downstream of the exit plane. Correlation was achieved by plotting G_0/G_{TH}, the ratio of the observed critical mass velocity to that determined from the homogeneous flow model vs. X_{TH}, the quality calculated from the homogeneous flow model. G_0/G_{TH} was not affected by changes in probe diameter, method of mixing, or by test section lengths greater than 9 in. but was decreased by the addition of a surface-action agent.

Reference: Faletti, Duane W., and R. W. Moulton, **A.I.Ch.E. Journal, 9,** No. 2, p. 247 (1963).

Key Words: Fluid Flow-8, Fluid Mechanics-8, Entrainment-8, Boiling-9, Bubbling-9, Separation-9, Efficiency-9, Trays-9, Velocities-6, Pressure-6, Voids-6, Quality-6, Entrainment-7, Loops-10, Boiling- , Dimensional Analysis-10.

Abstract: A simple model for carryunder phenomena (entrainment of vapor) is presented. The dominating factor in this analysis is the definition of a specific area in the riser from which carryunder emanates. Data taken from an atmospheric air-water loop are compared with the predicted values for weight percent carryunder for the parameter range studied: mixture qualities from 0.2×10^{-3} to 2.0×10^{-3}, and downcomer velocities from 1 ft./sec. to 2.5 ft./sec.

A dimensional analysis of carryunder has also been made and the results used to develop empirical correlations for predicting carryunder. A series of high-pressure data from a steam-water loop (pressures of 600, 1,000, and 1,500 lb./sq. in. downcomer velocities from 0.5 to 2.5 ft./sec. and void fractions from 0.1 to 0.5) is represented by an empirical dimensionless correlation. Finally, a nondimensionless empirical correlation has been developed and applied to the data for both loops. Some data obtained from a large reactor system are also presented.

Reference: Petrick, Michael, **A.I.Ch.E. Journal, 9,** No. 2, p. 253 (1963).

(*Continued on page 283*)

where K, b, d, and a are constants determined by experimentation and C is the temperature in degrees Centigrade.

Summaries of the data obtained with LOX are given in Table 1 and Figure 1. A summary of the final results pertaining to other liquids is given in Table 2.

This study of the flow of liquids in horizontal capillaries is only a brief phase of the problem of fluid movement under zero gravity. The data obtained cover only a small range of capillary radii and flow distances. Much work remains to be done in determining the effects of the variables upon the given constants, and also on materials with porous or fibrous structures. The work done in this study is sufficient to characterize the flow in short capillaries only.

ACKNOWLEDGMENT

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NOTATION

r = tube radius, cm., L

T = time, sec., T

Z = tube length, cm. L

ρ = density, gm./cc., $FL^{-4}T^2$

σ = surface tension, dynes/cm., FL^{-1}

μ = viscosity, gm./sec. cm., FTL^{-2}

C = temperature, °C.

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